Advancing Frontiers in Finite Element Procedures

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ABSTRACT: We focus briefly on some of our recent achievements to advance the frontiers of finite element procedures. The advances pertain to the development of methods for dynamic analyses, specifically for transient and frequency solutions, optimal low-order shell elements, a novel 8-node 3D solid element, and the development of the AMORE paradigm using overlapping finite elements.

1 INTRODUCTION

While the finite element method is now widely used in numerous applications of engineering and the sciences, it is also recognized that this use is bound to increase significantly. For example, there is large potential of a much broader application in computer-aided design and in the analysis of geometries obtained by digital scans, for a wide use in the fields of engineering and the medical sciences, for use with and in artificial intelligence, and in efforts of homeland security.

The foundations and fundamental procedures of finite element procedures are well established and have been published widely (see e.g. Bathe 2014a). However, the ease of use of finite element procedures should be much increased and more effective finite element procedures are greatly needed. Hence, research is performed to increase the effectiveness and efficiency of finite element schemes and to render the use of finite element procedures less human-intensive.

Some basic areas in which more efficient techniques are needed are the dynamic analyses of structures, specifically transient step-by-step solutions and frequency/ mode shape calculations, the analyses of shells and three-dimensional solids, and meshing techniques to alleviate the human effort required for a finite element simulation.

In this paper, we briefly review some of our recent research accomplishments to advance the finite element schemes in these areas. In our developments we focus on the reliability and efficiency of the procedures, and on their possible use in the analyses of complex structures. We apologize that due to lack of space we exclusively refer to our papers, in which however many references to other works are given.

2 ADVANCES FOR DYNAMIC ANALYSES

To achieve progress in the simulation of dynamic phenomena, we have increased significantly the efficiency of the subspace iteration method to calculate frequencies and mode shapes and enhanced an implicit time integration scheme.

2.1 *The enriched subspace iteration method*

The original subspace iteration scheme (Bathe 2014a) for the calculation of frequencies and mode shapes is widely used in engineering and the sciences. We focused our research on reaching a significant speed-up of the solution scheme by using the "turning of the iteration vectors" in each iteration step (Kim & Bathe 2017).

Figure 1 illustrates the speed-up of the solution scheme. An important ingredient is that the subspace iterations can be parallelized in SMP and DMP (Bathe 2013). Figure 2 depicts a typical speed-up in solution time using the schemes when the computations are parallelized.

The same concept can also be used to enrich the method further by using the "turning of the turning vectors", thus doubly enriching the scheme, reaching however, in general, not a further increase in speed as much as shown in Fig. 1 (Wilkins & Bathe 2019).

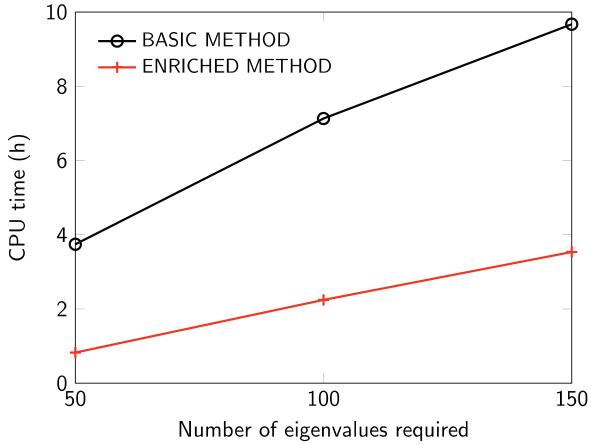


Figure 1 CPU time for calculating the smallest frequencies / mode shapes using the Bathe subspace iteration methods of a beam discretized by 8-node brick elements (number of degrees of freedom = 1,520,493, half-bandwidth = 507), single core Intel 2.4 GHz CPU, column solver, see Kim & Bathe 2017.

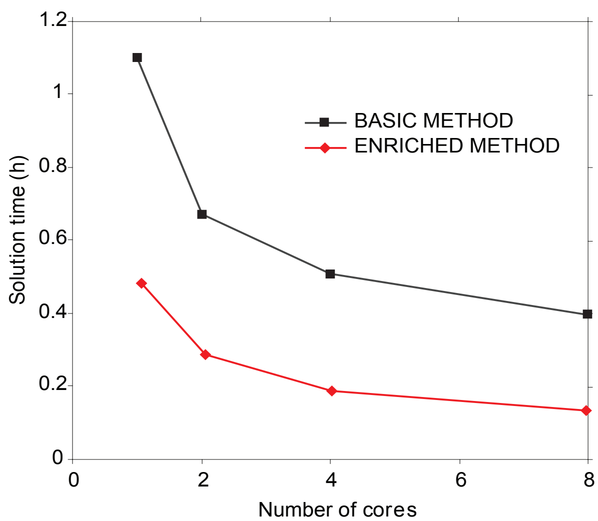


Figure 2 Time for calculating the lowest 100 frequencies / mode shapes of a bolted wheel model using the Bathe subspace iteration methods; number of degrees of freedom = 1,495,257 with 3,374 contact equations. ADINA results using Linux WS, Intel 3.2 GHz CPU.

2.2 *Generalizing the Bathe implicit time integration scheme*

The implicit time integration scheme proposed by Bathe (Bathe 2007) has been found to be effective in many linear and nonlinear analyses, see e.g. Kroyer, Nilsson & Bathe 2016. The procedure integrates accurately the response in the frequencies that should be integrated and cuts out spurious response, thus increasing the accuracy of solutions and also the convergence properties in the Newton-Raphson iterations of nonlinear analyses. A valuable asset of the scheme is that no parameters need be set (Bathe & Noh 2012, Noh & Bathe 2018). Although two sub-steps are used for each time step Δt, of course only the stability and cost of the *overall solution* decide whether the scheme is efficient. We also refer to Noh & Bathe 2013 for an explicit time integration scheme based on using two sub-steps for each time step.

To generalize the Bathe implicit time integration procedure, we proposed the β_1 / β_2 - Bathe time integration scheme (Malakiyeh, Shojaee & Bathe 2019). In this procedure the two parameters β_1 and β_2 can be set to obtain the originally proposed scheme and a method more effective in certain applications, specifically when the response in high frequencies should not be cut out and in wave propagation solutions. Figure 3 gives results and illustrates that, in this wave propagation analysis, as the CFL number = $c\,\Delta t / \Delta x$ (where c is the analytical wave speed and Δx the element length)

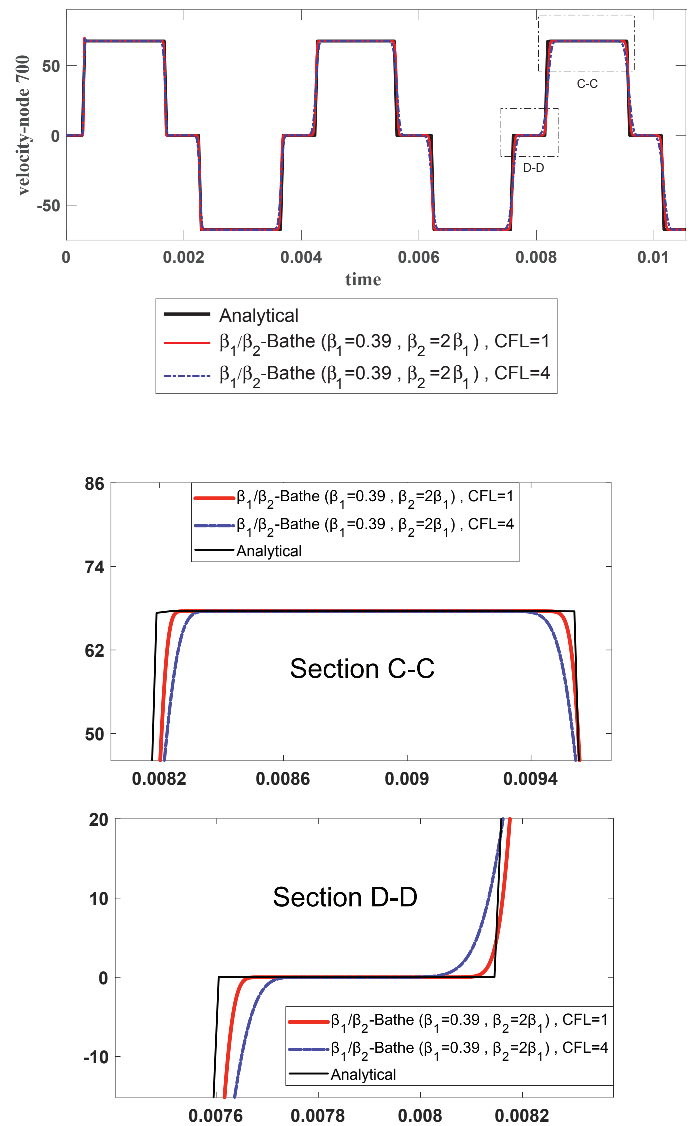


Figure 3 Solution of 1D wave propagation, clamped bar idealized by one thousand 2-node truss elements, β_1/β_2 - Bathe method.

is decreased, increasingly more accurate solutions are obtained. This is an important property for practical analyses.

Another approach is to use the spectral radius at very large time steps, ρ_∞, as a parameter, as in the ρ_∞- Bathe scheme (Noh & Bathe 2019). In this scheme only ρ_∞ is employed to choose the curves of the spectral radius and amplitude decay, see Fig. 4. The splitting ratio of the time step is chosen for optimal amplitude decay. This scheme and the β_1 / β_2-Bathe scheme encompass the trapezoidal rule of time integration as a special case.

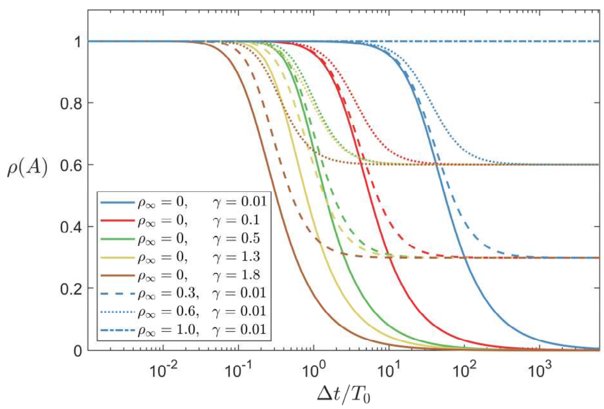


Figure 4 Spectral radii of approximation operator of the ρ_∞-Bathe method when $\xi = 0$ for various values of ρ_∞ ; γ denotes the splitting ratio of the time step and is set in the time stepping to be optimal for the selected ρ_∞.

3 ADVANCES FOR SHELL ANALYSES

Although we started to focus research efforts already decades ago on the development of shell elements, and the MITC4 shell element formulation (MITC is an acronym for "mixed interpolation of tensorial components" and the "4" refers to "4 nodes") proposed in 1984 is widely used in academia and commercial programs, we only recently reached a 4-node element, which shows an almost optimal behavior in membrane *and* bending-dominated conditions when using uniform *and* distorted meshes see Ko, Lee & Bathe 2017a.

Figure 5 shows the analysis results obtained in the analysis of a hyperboloid shell in membrane-dominated and in bending-dominated conditions. The MITC4+ element performs remarkably well even when distorted meshes are used. It is important to employ in the result evaluations an appropriate norm to measure the solution errors and we use the s-norm (Hiller & Bathe 2003). We also proposed a new triangular shell element, see Lee, Lee & Bathe 2014 and Jun, Yoon, Lee & Bathe 2018.

4 ADVANCES FOR ANALYSES OF TWO-AND THREE-DIMENSIONAL SOLIDS

Based on the MITC technique, we have recently also proposed a new 4-node element for two-dimensional and a new 8-node "brick element" for three-dimensional (3D) analyses of solids (Ko, Lee & Bathe 2017b and Ko & Bathe 2018a).

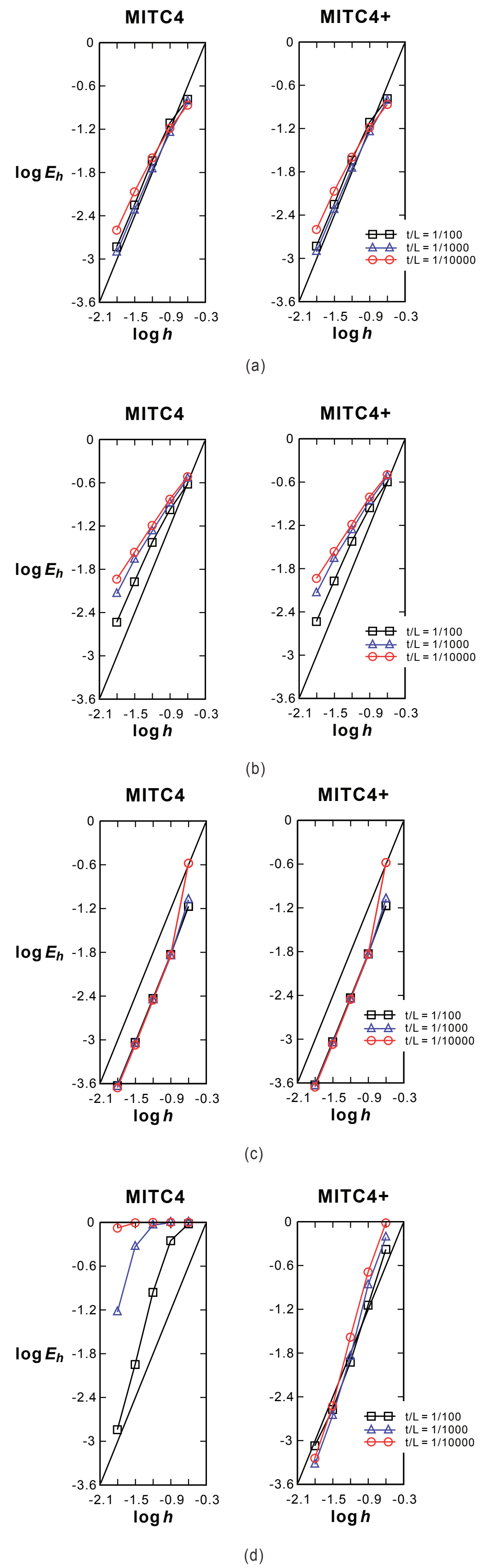


Figure 5 Convergence curves for hyperboloid shell problems; clamped membrane-dominated with (a) graded regular and (b) distorted meshes; free bending-dominated with (c) graded regular and (d) distorted meshes (see Ko, Lee & Bathe 2017a).

To formulate the 3D 8-node element we are using a thought experiment: we consider a stable truss structure with a minimum number of 2-node truss elements to have stability, see Fig. 6. The center of each truss element gives a point from which to interpolate the truss strain component over the 8-node brick element domain. A minimum number of truss elements is used to avoid locking, and a stabilization is activated in nonlinear analysis. The finally-reached brick element is stable and efficient and in geometrically nonlinear analyses does not show any hour-glassing, like do the elements using incompatible modes, see Ko & Bathe 2018a and Sussman & Bathe 2014, e.g. Fig. 7.

While the element formulation is largely based on physical reasoning, we also performed a numerical study of the inf-sup conditions regarding the element performance in possible shear and membrane locking, and locking in the analysis of incompressible media (Ko & Bathe 2018b). For the solution of incompressible media we use the u/p formulation (Bathe 2014a), see Fig. 7.

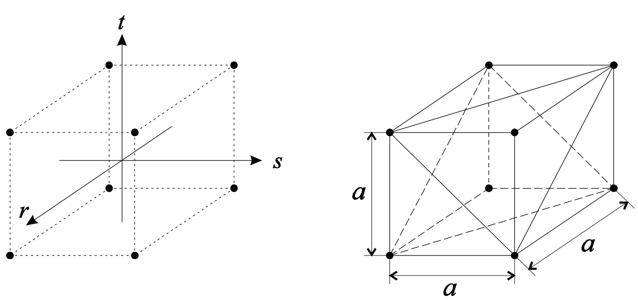
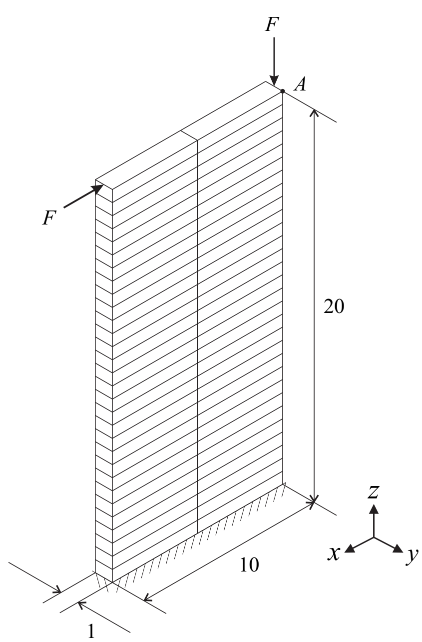


Figure 6 The 8-node brick element and its representation by a truss structure with a minimum number of 2-node truss elements to have stability of the truss structure.



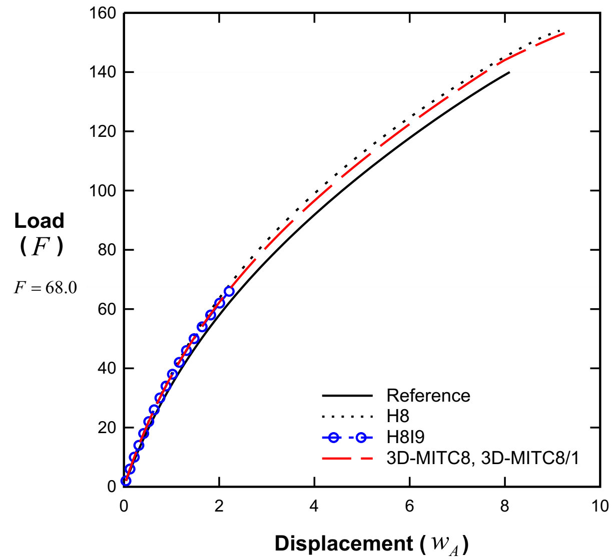
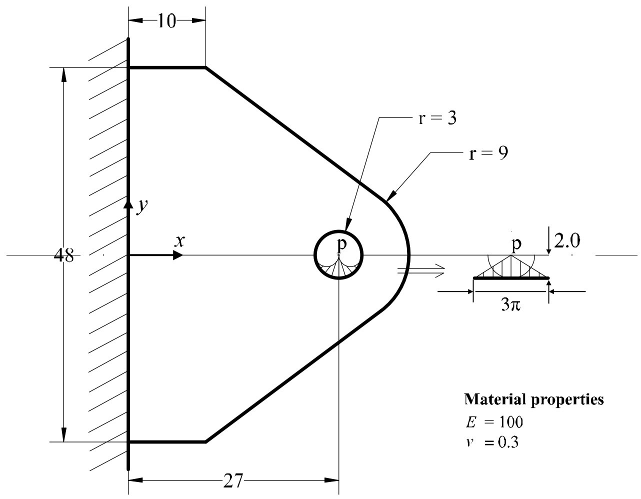


Figure 7 Cantilever model and solutions obtained, Poisson's ratio = 0.0; reference: using 27-node displacement-based element, H8: using 8-node displacement-based element, H8I9: using 8-node displacement-based element with incompatible modes, 3D-MITC8: using new element, 3D-MITC8/1: using new element with constant pressure interpolation.

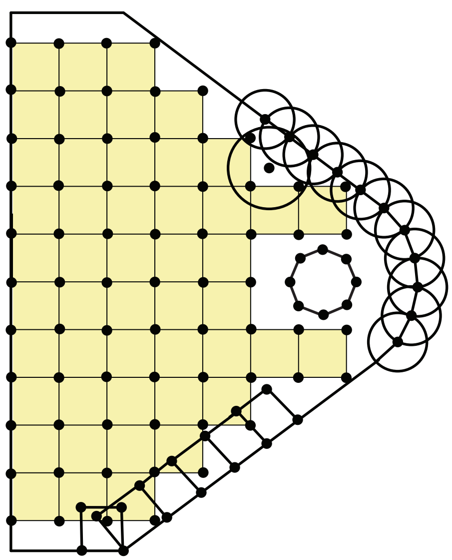
5 THE AMORE PARADIGM

In traditional finite element analysis, a major effort by the analyst is frequently required for meshing. To drastically reduce the time required for meshing, specifically when geometries in CAD or by 3D digital scans are available, we proposed the AMORE paradigm for Automatic Meshing with Overlapping and Regular Elements (Bathe 2016, Bathe & Zhang 2017 and Bathe 2019). The AMORE scheme shows much potential for general finite element analyses, see also Zhang & Bathe 2017, Zhang, Kim & Bathe 2018, and Kim, Zhang & Bathe 2018.

The meshing steps in the AMORE scheme of analysis are, to immerse the part to be analyzed in a grid of cells, Cartesian or other grid that could be refined in some areas, to discretize the boundary, remove all cells that lie outside this boundary or cut the boundary, turn all remaining cells into regular finite elements, and, finally, to cover the then unfilled space of the domain with overlapping finite elements, see Fig. 8. An important point is that these overlapping elements are distortion-insensitive. Hence the algorithm aims to give non-distorted regular elements, which perform optimally because they are undistorted, and overlapping elements that can be highly distorted and still perform well. The overlapping finite elements are formulated using the concepts of meshless methods, see e.g. Lai & Bathe 2016 and Nicomedes, Bathe, Moreira & Mesquita 2017, and are also related to the use of interpolation covers, see Kim & Bathe 2014.



(a)



(b)

Figure 8 AMORE shown schematically for the analysis of a bracket; (a) geometry and (b) mesh used showing schematically only some overlapping elements.

Figure 9 shows some analysis results of a wave propagation solution in which all elements are overlapping (polygonal elements overlap to form triangular elements, see Bathe 2019) and in addition to the usual polynomials also trigonometric functions are used as degrees of freedom, see Kim, Zhang & Bathe 2018. Here note that accurate solutions are difficult or impossible to obtain of complex wave propagations using traditional finite elements.

Figure 10 shows the analysis of a cantilever plate with holes. Here too, the overlapping elements are polygonal elements which overlap in triangular regions, resulting in the formulation of triangular elements that are distortion-insensitive. For the analysis results obtained see Zhang, Kim & Bathe 2018 or Bathe 2019.

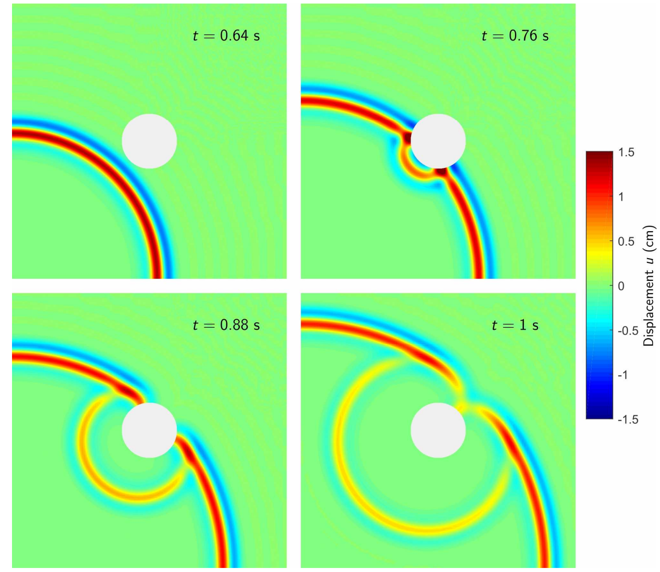


Figure 9 Snapshots of transverse displacement distributions of a quarter of a prestressed membrane with symmetrically-placed circular holes at various observation times calculated using AMORE.

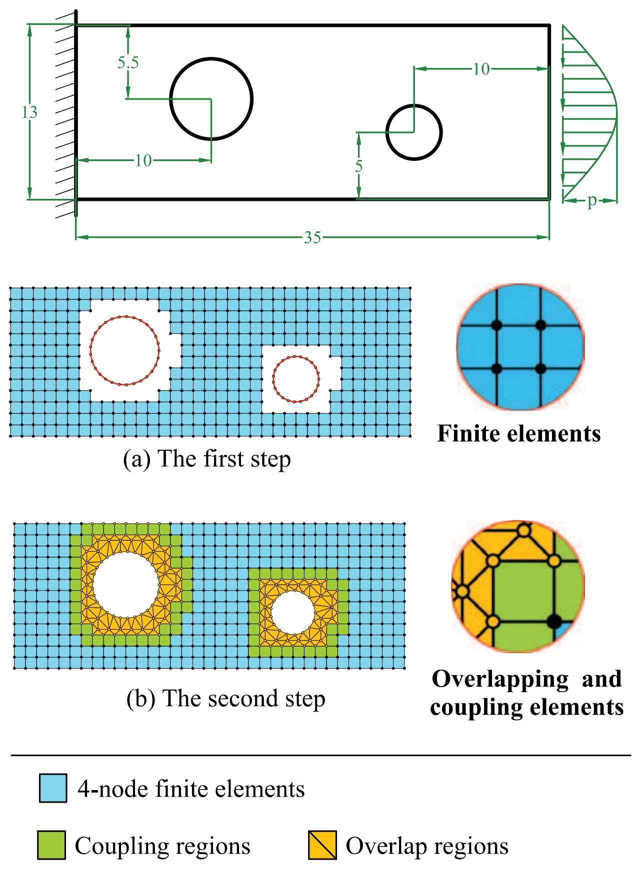


Figure 10 AMORE mesh used in the analysis of a plate; (a) Cartesian cells with some cells removed (b) final mesh.

6 CONCLUDING REMARKS

Based on our experience in finite element methods we expect that the use of finite element methods will increase very significantly and research advances are still much needed. In this presentation we briefly surveyed some of our latest achievements in advancing finite element methods for general analysis.

Some research areas that we did not comment on are the huge fields of analysis of fluids and multiphysics problems involving solids, fluids and electro-magnetic effects (see, regarding some of our achievements, Bathe, Zhang & Yan 2014 and Bathe 2014b) and the analysis of proteins and DNA structures, see e.g. Bathe 2014b and Sedeh, Yun, Lee, Bathe & Kim 2018. Many advances are still needed in these fields and the AMORE scheme is, here too, a promising approach to explore.

7 ACKNOWLEDGEMENTS

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